THEORY OF AUTOMATA & FORMAL LANGUAGES

DFA: Counting and Modular Constraints

Remember counts using a finite number of states

Count → Update → Accept → Verify

Counting with finite states

Throughout this lecture, $\Sigma = \{a,b\}$; count occurrences of a.

What must be remembered?

Exact count up to a fixed threshold.

Parity: even or odd.

Remainder after division by m.

Reading b never changes the count.

Start with zero occurrences.

Choose states from the condition

Exactly 2: counts 0, 1, 2, >2.

At least 2: counts 0, 1, ≥2.

Even: remainders 0, 1 mod 2.

Divisible by 3: remainders 0, 1, 2.

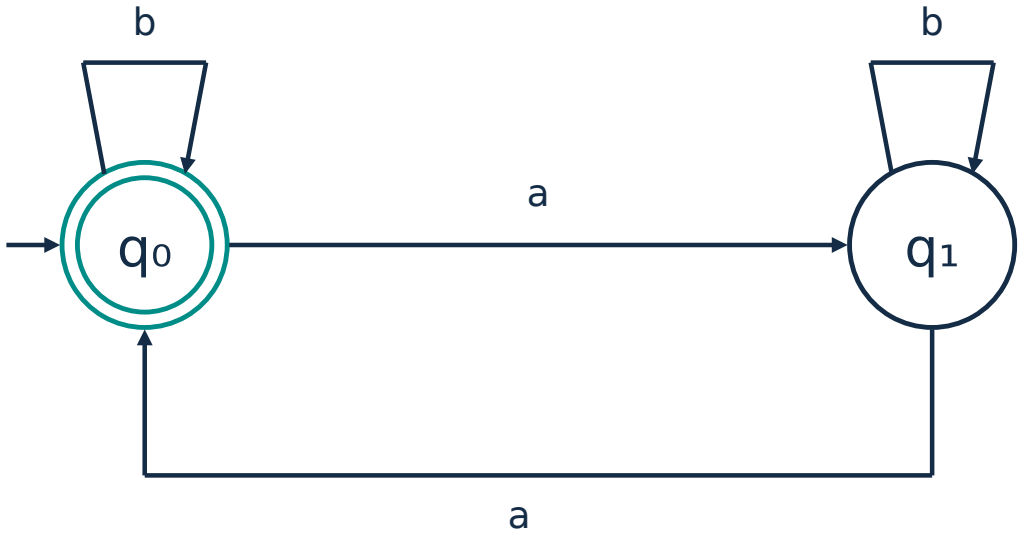
Mark the states meeting the rule.

The machine remembers only information needed to decide membership.

Even and odd numbers of a

$L = \{w \in \Sigma^* : |w|_a \bmod 2 = 0\}$.

Complete DFA



Transition table and trace

State	a	b
→★ q_0	q_1	q_0
q_1	q_0	q_1

aba: $q_0 \to q_1 \to q_1 \to q_0$

Accept: aba, ε; reject: ab.

For odd counts, keep the same transitions and make only q_1 accepting.

Exactly two occurrences of a

$L = \{w \in \Sigma^* : |w|_a = 2\}$.

State meanings

q_0: zero a (initial).

q_1: exactly one a.

q_2: exactly two a (final).

q_3: three or more a (dead).

Reading b preserves every count.

Transition table

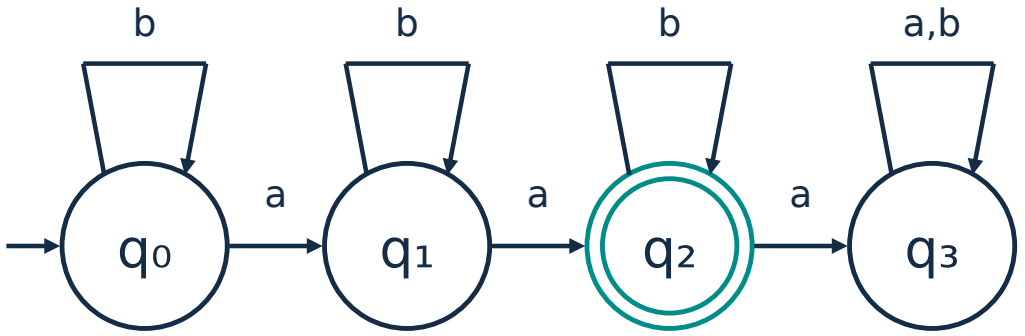
State	a	b
→ q_0	q_1	q_0
q_1	q_2	q_1
★ q_2	q_3	q_2
q_3	q_3	q_3

Once the count exceeds two, it cannot decrease.

Exactly two: draw and verify

The fourth state is necessary to reject excessive counts.

Complete DFA



Manual verification

aba: $q_0 \to q_1 \to q_1 \to q_2$

Result: accept (two a).

aaa: $q_0 \to q_1 \to q_2 \to q_3$

Result: reject (three a).

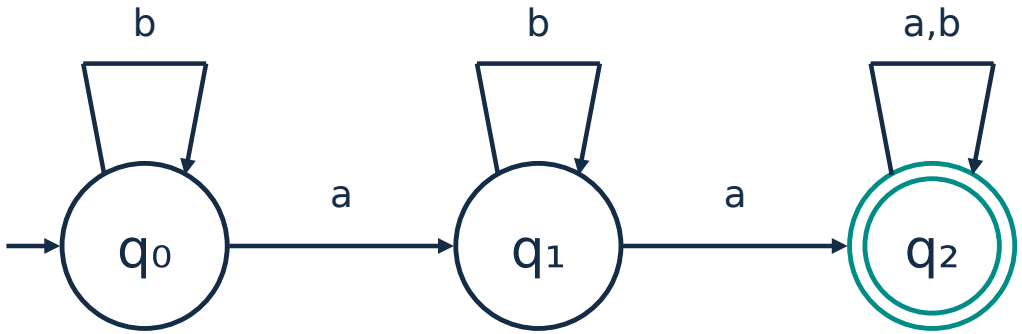
ε and ab: rejected.

Reaching q_2 early does not guarantee acceptance.

At least two occurrences of a

$L = \{w \in \Sigma^* : |w|_a \geq 2\}$.

Complete DFA



Transition table and trace

State	a	b
→ q_0	q_1	q_0
q_1	q_2	q_1
★ q_2	q_2	q_2

aaa: $q_0 \to q_1 \to q_2 \to q_2$: accept

Here q_2 is an accepting sink, not a dead state.

Divisibility by three: mathematical model

$L = \{w \in \Sigma^* : |w|_a \bmod 3 = 0\}$.

Remainder states

q_0: remainder 0 (initial, final).

q_1: remainder 1.

q_2: remainder 2.

$\delta(q_r,a) = q[(r + 1) \bmod 3]$

$\delta(q_r,b) = q_r$

Transition table

State	a	b
→★ q_0	q_1	q_0
q_1	q_2	q_1
q_2	q_0	q_2

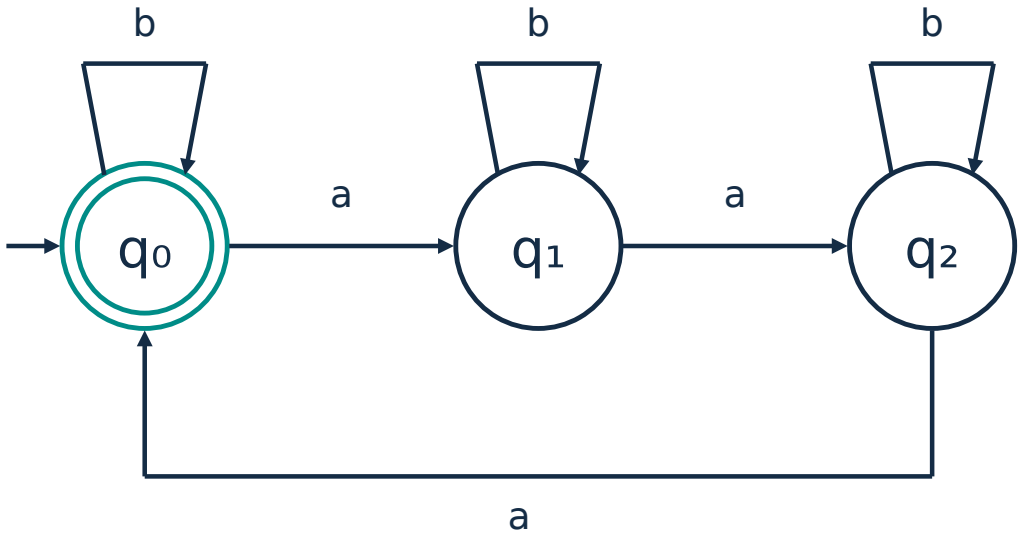
Zero is divisible by three.

Only the remainder matters; the actual count may grow without limit.

Divisibility by three: draw and verify

The a-transitions form a cycle; the b-transitions are loops.

Complete DFA



Manual verification

aaba: $q_0 \to q_1 \to q_2 \to q_2 \to q_0$

Result: accept (three a).

aab: $q_0 \to q_1 \to q_2 \to q_2$

Result: reject (two a).

ε and aaaaaa: accepted.

For divisor m, use m remainder states and accept remainder zero.

Practice and comparison

Select the state meanings before drawing any arrows.

Choose the correct model

Exactly n: n + 2 states.

At least n: n + 1 states.

Divisible by m: m states.

Odd count: final remainder 1.

At most 2: accept q_0, q_1, q_2.

Check your answers

ε: even and divisible by 3.

aaa: at least 2, not exactly 2.

aab: exactly 2 and even.

aaaa: even, not divisible by 3.

All examples count a, not length.

Next: DFA construction for combined conditions using product states.