THEORY OF AUTOMATA & FORMAL LANGUAGES

DFA Minimization Table-Filling Method

Identify distinguishable pairs and merge equivalent states.

Remove unreachable states → Mark pairs → Merge → Verify

Why fill a table?

Compare every unordered pair of reachable states.

Meaning of a marked pair

A pair (p,q) is distinguishable if some suffix w separates their acceptance.

If exactly one state accepts ε, mark the pair immediately.

A marked pair must never be merged.

An unmarked pair is a candidate for merging.

Only after convergence is an unmarked pair equivalent.

Systematic algorithm

1. Remove unreachable states.
2. Draw a triangular table of state pairs.
3. Mark accepting/non-accepting pairs.
4. Propagate marks using transitions.
5. Merge unmarked pairs after no new marks.

Use a complete DFA; add a sink first if transitions are missing.

Worked example — original DFA

Alphabet Σ = {a,b}; start A; accepting set F = {E}.

Complete transition table

State	a	b
→ A	B	A
B	C	B
C	E	D
D	E	C
* E	E	E

Reachability and remaining row

A is reached by ε; B by a.

C is reached by aa; D by aab.

E is reached by aaa.

All five states are reachable.

The machine accepts strings containing at least three a symbols.

Step 1 — draw the empty table

Five states give 5 × 4 / 2 = 10 unordered pairs.

Initial triangular table

	A	B	C	D
B	·			
C	·	·		
D	·	·	·	
E	·	·	·	·

How to read the table

Each cell represents one unordered pair.

Row D, column B means (B,D).

No diagonal: a state equals itself.

Do not duplicate (A,B) and (B,A).

Dot (·) means not yet marked.

A blank or dot is not proof of equivalence until the algorithm stops.

Step 2 — initial marking (round 0)

Mark pairs where exactly one state belongs to F = {E}.

Table after round 0

	A	B	C	D
B	·			
C	·	·		
D	·	·	·	
E	0	0	0	0

Reason for each mark

(A,E), (B,E), (C,E), (D,E)

A, B, C, D reject the empty string.

E accepts the empty string.

Therefore ε distinguishes all four pairs.

0 means marked in round 0.

Four pairs marked; six pairs remain unmarked.

Step 3 — propagation (round 1)

Use only the marks present at the end of round 0 for this teaching example.

Table after round 1

	A	B	C	D
B	·			
C	1	1		
D	1	1	·	
E	0	0	0	0

Check transitions on a

(A,C) → (B,E): marked 0

(A,D) → (B,E): marked 0

(B,C) → (C,E): marked 0

(B,D) → (C,E): marked 0

Thus a distinguishes these four pairs.

Rule: mark (p,q) if some input takes it to an already marked pair.

Step 4 — propagation (round 2)

Recheck every remaining unmarked pair using round 1 marks.

Table after round 2

	A	B	C	D
B	2			
C	1	1		
D	1	1	·	
E	0	0	0	0

Check the last two pairs

(A,B) on a → (B,C): marked 1.

Thus aa distinguishes A and B.

(C,D) on a → (E,E): identical.

(C,D) on b → (D,C): same pair.

Round 3 adds no marks: stop.

Only (C,D) remains unmarked. Therefore C and D are equivalent.

Step 5 — construct the minimum DFA

Merge C and D into X; keep A, B and E separate.

New transition table

State	a	b
→ A	B	A
B	X	B
X	E	X
* E	E	E

State meanings

A: no a has been read.

B: exactly one a has been read.

X = {C,D}: exactly two a symbols.

E: at least three a symbols.

Four reachable, distinguishable states.

For X, either representative C or D leads to E on a and X on b.

Verification and practice

The minimum DFA recognizes the same language as the original.

Manual string verification

ε: A → reject in both machines.

aa: A → B → X (original C): reject.

aab: A → B → X → X (original D): reject.

aaba: A → B → X → X → E: accept.

bbaba: two a symbols → reject.

Questions and answers

Why is (C,D) unmarked? Same future acceptance.

What distinguishes A and B? aa.

What distinguishes B and C? a.

What distinguishes A and E? ε.

Can marked pairs be merged? No.

Five states become four; all nine marked pairs are distinguishable.