Mealy Machine

Finite-state machines with transition-based output

From Moore machines to input-dependent output

Lecture focus: definition • representation • design • tracing

Learning outcomes

01 Define
Identify the six components of a Mealy machine.

02 Represent
Read input/output labels and transition tables.

03 Trace
Compute the output sequence for a given input.

04 Design
Construct parity and suffix-detection machines.

Moore vs Mealy: the key distinction

Feature	Moore	Mealy
Output depends on	Current state	State and input
Diagram labels	State / output	Input / output on edge
Output length (n inputs)	n + 1 including initial	n
Initial output	Defined	None before input

Both use deterministic transitions; Mealy outputs are produced on transitions.

Formal definition

$M = (Q, \Sigma, \Gamma, \delta, \lambda, q_0)$

Q: finite states; Σ: input alphabet; Γ: output alphabet.

$\delta : Q \times \Sigma \to Q$

Transition function maps current state and input to next state.

$\lambda : Q \times \Sigma \to \Gamma$

Output function maps current state and input to output symbol.

$q_0 \in Q$

Initial state; no final-state set is required for a transducer.

How a Mealy machine processes input

Step 1 — Read

Start in q_0 and read the next input symbol a.

Step 2 — Produce

Emit b = λ(q, a) using the current state q.

Step 3 — Move

Update q ← δ(q, a); repeat until input is exhausted.

Remember

For n input symbols, exactly n output symbols are generated.

Example 1: even parity of 1s

Requirement

Emit 1 when the number of 1s read so far is even; otherwise emit 0.

States

E: even number of 1s (initial); O: odd number of 1s.

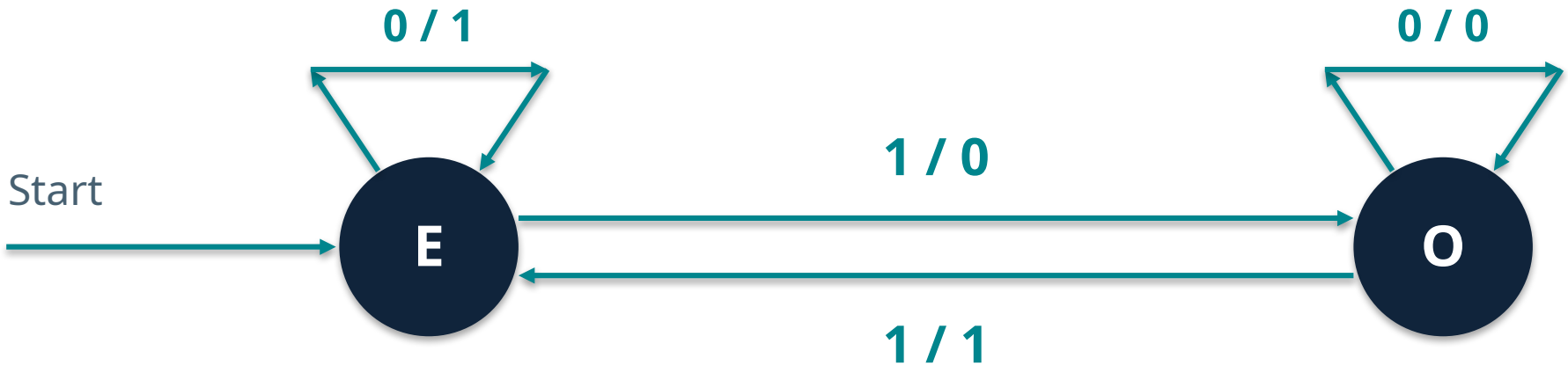
Input 0

Parity is unchanged; emit the current parity indicator.

Input 1

Parity toggles; emit the indicator for the new parity.

Example 1: state diagram



Each transition is labeled input / output. Output describes parity after reading the symbol.

Example 1: transition/output table

Current state	Input 0: next/output	Input 1: next/output
→ E	E / 1	O / 0
O	O / 0	E / 1

A cell combines δ(q, a) and λ(q, a). The arrow marks the initial state.

Example 1: trace input 10110

Step	State before	Input	State after	Output
1	E	1	O	0
2	O	0	O	0
3	O	1	E	1
4	E	1	O	0
5	O	0	O	0

Output: 00100. There is no separate initial output in a Mealy machine.

Example 2: detect suffix 01

Requirement

Emit 1 whenever the input read so far ends in 01; otherwise emit 0.

State A (initial)

No trailing 0: input is empty or the last symbol was 1.

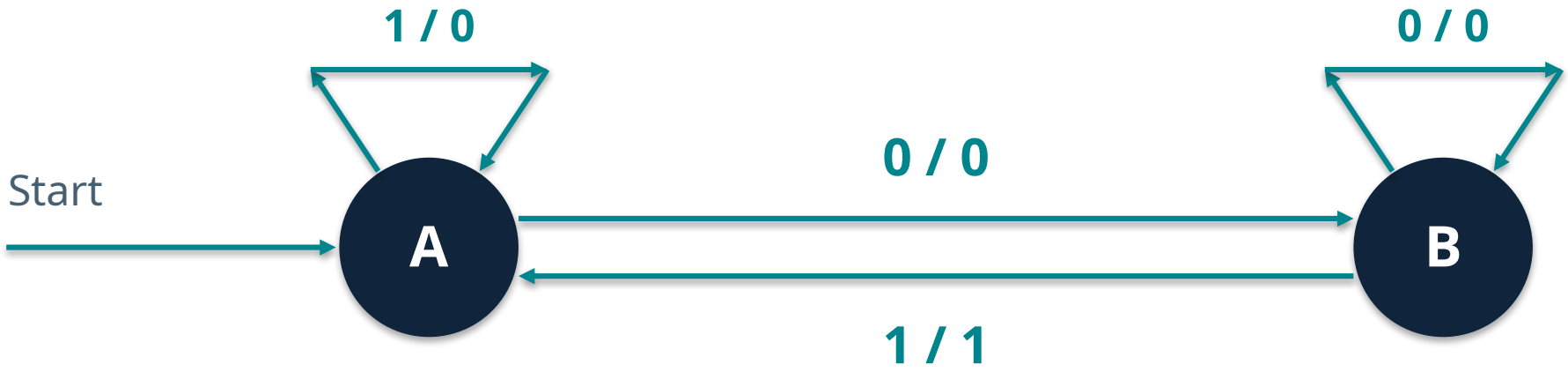
State B

Last input symbol was 0.

Key transition

B on input 1 emits 1 and returns to A; all other transitions emit 0.

Example 2: state diagram



Only the transition B → A on input 1 recognizes the suffix 01.

Example 2: transition/output table

Current state	Input 0: next/output	Input 1: next/output
→ A	B / 0	A / 0
B	B / 0	A / 1

The Mealy machine needs two states for this task; the Moore design in Lecture 23 used three.

Example 2: trace input 01001

Step	State before	Input	State after	Output
1	A	0	B	0
2	B	1	A	1
3	A	0	B	0
4	B	0	B	0
5	B	1	A	1

Output: 01001. The two 1s mark the ends of occurrences of 01.

Common mistakes

Confusing output placement

In Mealy machines, write output on transitions, not inside states.

Using the wrong state

Evaluate λ(q, a) using the state before the transition.

Adding an initial output

An empty input produces an empty output in the standard Mealy model.

Assuming fewer states always

Mealy designs can use fewer states, but this is not guaranteed.

Practice questions

Q1
Trace the parity machine for input 11001.

Q2
Trace the suffix-01 machine for input 001011.

Q3
Design a one-state Mealy machine that complements every input bit.

Q4
Explain why the output length differs from the Moore convention.

Solutions and recap

A1 — 01110

Parity after each input: odd, even, even, even, odd.

A2 — 001010

The suffix 01 occurs at positions 3 and 5.

A3 — One state

Self-loops labeled 0 / 1 and 1 / 0.

A4 — Transition output

Mealy emits once per input; Moore also emits from the initial state.