Mealy to Moore Conversion

Moving outputs from transitions to states

State splitting, initial outputs and verification

Lecture focus: rule • algorithm • examples • verification

Learning outcomes

01 Recognize

Identify destinations receiving transitions with different outputs.

02 Construct

Create one Moore state for each destination/output pair.

03 Verify

Compare output sequences after excluding the initial Moore output.

04 Practice

Convert machines using transition tables.

Mealy vs. Moore: a quick revision

Mealy machine

Output function $\lambda: Q \times \Sigma \to \Gamma$; edge labels are input/output.

Moore machine

Output function $\mu: Q \to \Gamma$; state labels are state/output.

Key challenge

A destination may receive edges with different outputs.

Output length

For n inputs, Mealy gives n outputs; Moore gives n + 1 including its initial output.

Formal conversion rule

Create destination/output pairs

$P = \{(\delta(q,a), \lambda(q,a)) : q \in Q, a \in \Sigma\}$.

Assign state output

$\mu((q,b)) = b$.

Define transitions

$\Delta((q,b),a) = (\delta(q,a), \lambda(q,a))$.

Create a fresh initial state I

$\mu(I) =$ – (dummy output; extend Γ); $\Delta(I,a) = (\delta(q_0,a), \lambda(q_0,a))$.

Conversion algorithm

Step 1

List every distinct output on transitions entering each state.

Step 2

Split each destination into one copy for each incoming output.

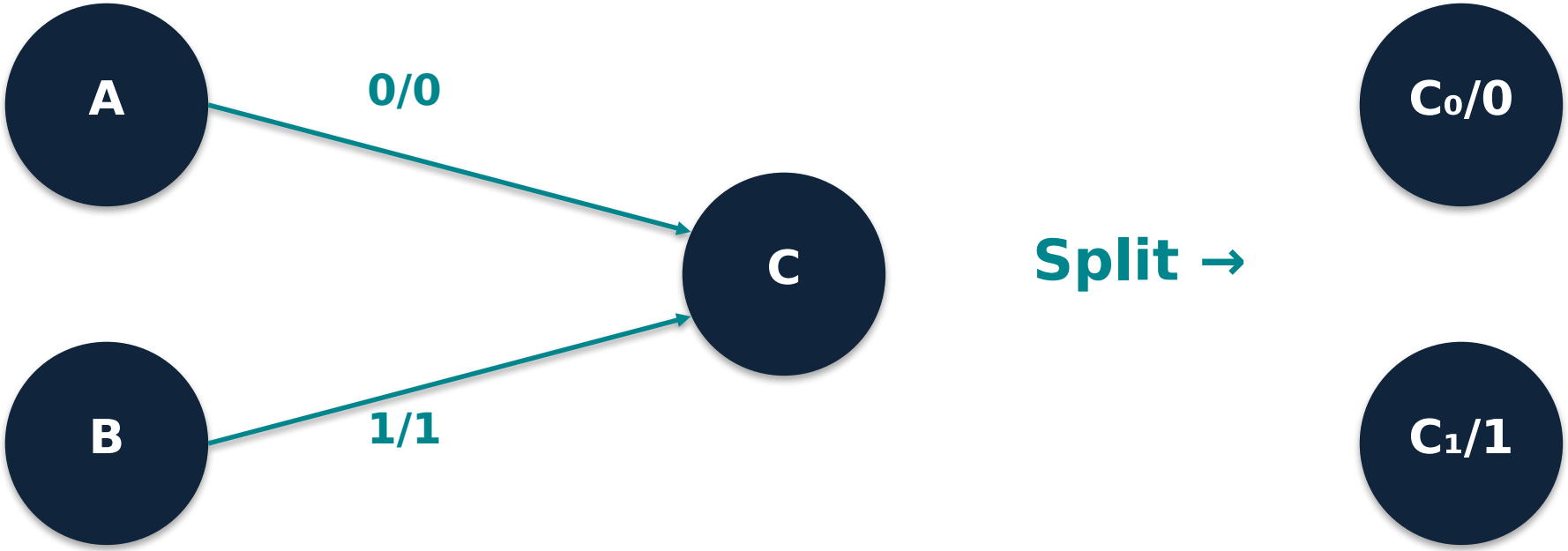
Step 3

Copy outgoing behavior from the original state to every copy; target the correct pair.

Step 4

Add the initial state, remove unreachable states, and test an input string.

Why state splitting is necessary



A single Moore state cannot represent both outputs. Create C_0 and C_1.

Example 1: original Mealy machine

State	Input 0	Input 1
→ A	A/0	A/1

A one-state Mealy machine echoes each input bit.

Example 1: identify the new states

Destination	Incoming outputs	Moore copies
A	0, 1	$A_0/0$, $A_1/1$
Initial	–	I/–

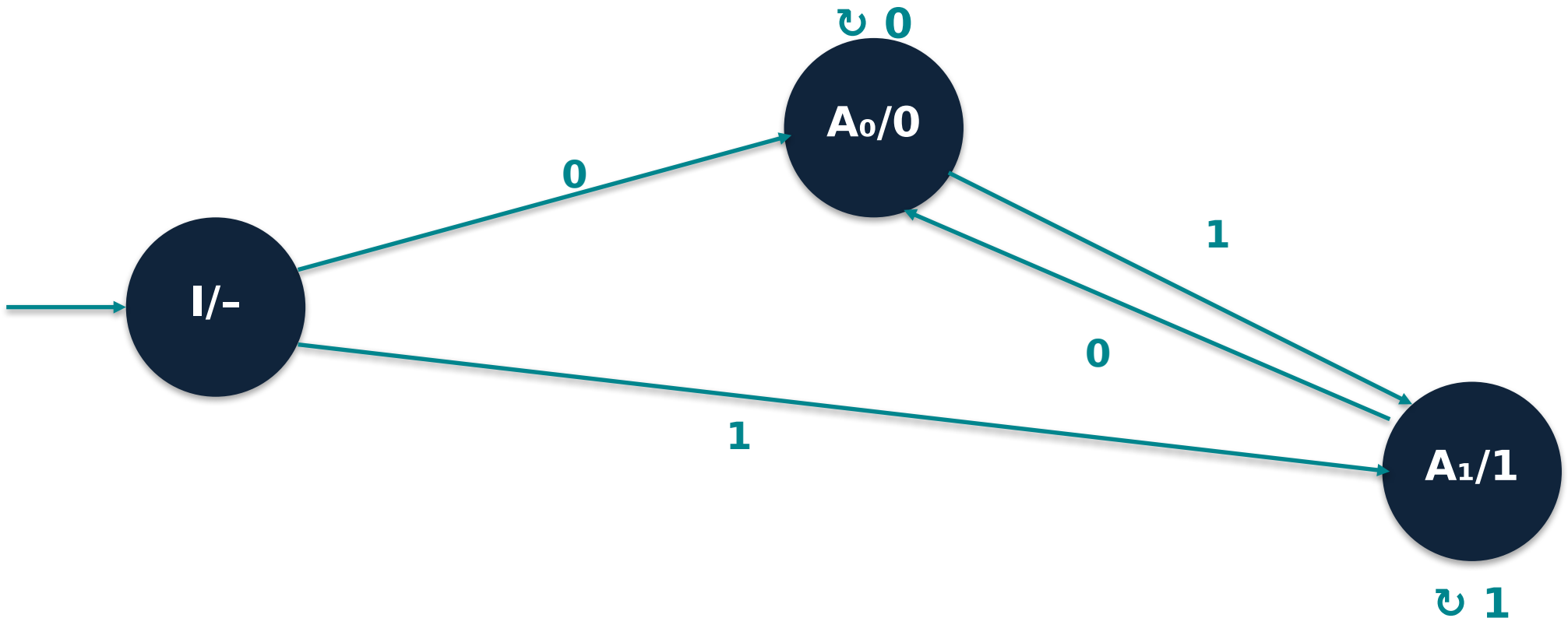
Both transitions enter A, but their outputs differ. Split A into two copies.

Example 1: converted Moore table

State	Input 0	Input 1	Output
→ I	A_0	A_1	–
A_0	A_0	A_1	0
A_1	A_0	A_1	1

Each copy inherits the same outgoing behavior as the original state A.

Example 1: converted state diagram



Loop notation: ↻ means a self-transition on the indicated input.

Example 1: verify input 1010

Step	Input	Mealy output	Moore state	Moore output
0	–	–	I	–
1	1	1	A_1	1
2	0	0	A_0	0
3	1	1	A_1	1
4	0	0	A_0	0

Mealy: 1010. Moore: –1010. Ignore the initial dummy output.

Example 2: detect suffix 01

State	Input 0	Input 1
→ A	B/0	A/0
B	B/0	A/1

A means no trailing 0; B means trailing 0. Output 1 only when 01 is completed.

Example 2: converted Moore table

State	Input 0	Input 1	Output
→ I	B_0	A_0	–
A_0	B_0	A_0	0
A_1	B_0	A_0	1
B_0	B_0	A_1	0

A receives outputs 0 and 1; B receives only 0. Thus A splits, but B does not.

Example 2: verify input 00101

Step	Input	Mealy output	Moore state	Moore output
0	–	–	I	–
1	0	0	B_0	0
2	0	0	B_0	0
3	1	1	A_1	1
4	0	0	B_0	0
5	1	1	A_1	1

Mealy: 00101. Moore: –00101. The traces match after the initial output.

Practice questions

Q1

Convert the one-state Mealy machine A: 0 → A/1, 1 → A/0.

Q2

For Q1, trace input 0110 in both machines.

Q3

A destination receives outputs 0, 1 and 1. How many copies are required?

Q4

For k original states and m output symbols, give an upper bound including I.

Solutions and recap

A1

I/–, A_0/0, A_1/1. From each state: 0 → A_1, 1 → A_0.

A2

Mealy: 1001. Moore: –1001.

A3 and A4

Two copies; at most km + 1 states before removing unreachable states.

Remember

Split by incoming output, copy outgoing behavior, and account for the initial output.