Solved Examination Questions

Final course lecture

Theory of Automata and Formal Languages • Lecture 45 of 45

Purpose

Apply definitions, constructions and proof techniques under examination conditions.

Format

Original examination-style questions with worked solutions and a final revision checklist.

Practice paper: suggested marking

Section A • 20 marks

Five short questions × 4 marks: models, outputs, closure, normal forms and complexity.

Section B • 40 marks

Four construction questions × 10 marks: DFA, NFA, CFG and PDA.

Section C • 40 marks

Four proof/design questions × 10 marks: pumping lemmas, TM and undecidability.

Q1–2 • Models and transducers (8 marks)

Q1. Compare DFA and NFA.

DFA: exactly one next state per state/symbol. NFA: a set of possible next states; acceptance requires at least one accepting path.

Q2. Compare Moore and Mealy output.

Moore: output depends on state; Mealy: output depends on state and input. For input length n, standard outputs have lengths n+1 and n.

Marking focus

Give formal transition/output distinctions and explain equivalence of DFA and NFA in language recognition.

Q3–5 • Short theory answers (12 marks)

Q3. Are CFLs closed under intersection?

Not in general. They are closed under intersection with a regular language.

Q4. State Chomsky Normal Form.

A → BC or A → a; allow a protected start rule $S_0 \to \varepsilon$ when ε belongs to the language.

Q5. Differentiate NP-hard and NP-complete.

NP-hard: every NP problem reduces to it in polynomial time. NP-complete: NP-hard and in NP.

Q6 • Construct a DFA (10 marks)

Question

Design a DFA over {0,1} accepting strings ending in 01.

State meanings

A: no useful suffix; B: suffix 0; C: suffix 01. Start A; only C accepts.

Transition table

A: 0→B, 1→A B: 0→B, 1→C C: 0→B, 1→A

Trace 1101: A → A → A → B → C (accept).

Q6 • DFA state diagram and justification

Invariant

Each state records the longest suffix of the input that is also a prefix of 01.



Other edges: A ↺ 1; B ↺ 0; C → B on 0; C → A on 1

Marking focus

State meanings (3), transitions (4), start/final states (2), trace (1).

Q7 • NFA to DFA (10 marks)

Question

NFA: start p, final q; $\delta(p,0)=\{p,q\}$, $\delta(p,1)=\{p\}$, $\delta(q,0)=\varnothing$, $\delta(q,1)=\{q\}$.

Subset construction

$A=\{p\}$ (start), $B=\{p,q\}$ (final).

A: $0\to B$, $1\to A$. B: $0\to B$, $1\to B$.

Result

The DFA accepts strings containing at least one 0. No other subset is reachable. Mark reachable subsets and final subsets explicitly.

Q8 • CFG and derivation (10 marks)

Question

Construct a CFG for $L=\{a^n b^n : n \geq 0\}$ and derive aaabbb.

Grammar and derivation

$S \rightarrow aSb \mid \varepsilon$

$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aaaSbbb \Rightarrow aaabbb$

Justification

Each recursive application adds one a on the left and one b on the right. The ε-rule terminates; n applications generate exactly $a^n b^n$.

Q9 • PDA for equal blocks (10 marks)

Question

Design an NPDA accepting $\{a^n b^n : n \geq 0\}$ by final state. Stack top is written first; Z is the bottom marker.

Transitions (push phase p, pop phase r)

$(p,a,Z) \to (p,AZ)$; $(p,a,A) \to (p,AA)$; $(p,\varepsilon,X) \to (r,X)$, $X \in \{A,Z\}$.

$(r,b,A) \to (r,\varepsilon)$; $(r,\varepsilon,Z) \to (f,Z)$.

Trace for aabb

$(p,aabb,Z) \Rightarrow (p,abb,AZ) \Rightarrow (p,bb,AAZ) \Rightarrow (r,bb,AAZ)$

$\Rightarrow (r,b,AZ) \Rightarrow (r,\varepsilon,Z) \Rightarrow (f,\varepsilon,Z)$. Accept only with input exhausted.

Q10 • Regular pumping lemma (10 marks)

Question

Prove $L=\{0^n1^n : n\geq 0\}$ is not regular.

Assume regular; choose witness

Let p be its pumping length and $w=0^p1^p$. For every valid split $w=xyz$, $|xy|\leq p$ and $|y|>0$, so $y=0^k$, $1\leq k\leq p$.

Pump down

$xz=0^{p-k}1^p$ is not in L. This contradicts the required membership for i=0.
Therefore L is not regular.

Q11 • CFL pumping lemma (10 marks)

Question

Prove $L=\{a^n b^n c^n : n \geq 0\}$ is not context-free.

Choose witness and analyze all splits

Assume CFL; $w=a^p b^p c^p$. For $w=uvxyz$, $|vxy| \leq p$ and $|vy|>0$. The window meets at most two of the three blocks.

Pump down

Deleting v and y reduces at least one symbol count, while at least one block keeps count p. Counts cannot all be equal. Contradiction.

Q12 • Turing machine design (10 marks)

Question

Describe a deterministic TM deciding $\{a^n b^n : n \geq 0\}$.

Algorithm

Verify input has form a*b*. Repeatedly mark the leftmost unmarked a as X, scan right to mark an unmarked b as Y, then return left.

Halting and correctness

Reject if a matching b is missing. When no a remains, accept iff no unmarked b remains. Each pass marks a new a, so the machine halts.

Q13 • Undecidability proof (10 marks)

Question

Use diagonalization to prove A_TM={<M,w> : M accepts w} is undecidable.

Assume a decider H exists

Construct D on input <M>: run H(<M,M>); reject if H accepts, otherwise accept. D halts on every valid machine encoding.

Contradiction

Run D on <D>. It accepts iff H says D does not accept <D>, i.e., iff D does not accept. No such H exists.

Bonus • DFA minimization

Question

A,B nonfinal; C final. Start A. On 0: A→B, B→A, C→C. On 1: A→C, B→C, C→C.

Solution

Initial partition {A,B} | {C}. A and B have identical destination blocks on both symbols; no refinement is needed.

Minimal DFA

U={A,B} start/nonfinal; V={C} final. U: 0→U, 1→V; V: 0→V, 1→V. Language: strings containing at least one 1.

Bonus • Regular expression conversion

Question

Find a regular expression for binary strings ending in 01.

Solution

(0|1)*01. The prefix is arbitrary; the final two symbols must be 01.

Reverse construction

An NFA can loop on 0 and 1 at the start, nondeterministically take a 0 to an intermediate state, then 1 to the final state.

Bonus • Normal forms and ambiguity

CNF conversion

S→aAB; A→a; B→b. Introduce T→a and U→AB, then S→TU. All productions now have CNF form.

Ambiguity witness

E→E+E | E*E | id. The word id+id*id admits both (id+id)*id and id+(id*id).

Resolution

E→E+T | T; T→T*F | F; F→id. Multiplication has higher precedence; both operators associate left.

Bonus • Rice, PCP and complexity

Rice’s theorem

Whether L(M) is finite is undecidable: it is a nontrivial semantic property of recognizable languages.

PCP example

Tiles (a,ab) and (ba,a): sequence 1,2 gives top aba and bottom aba.

Reduction direction

To prove a new problem B NP-hard, reduce a known NP-hard problem A to B in polynomial time. To prove NP-completeness, also show $B \in NP$.

Final comparison • Language classes

Regular ⊊ context-free

FA recognize regular languages; NPDA recognize CFLs. Witness separating them: $\{a^n b^n\}$.

Context-free ⊊ context-sensitive

Nondeterministic LBA recognize context-sensitive languages. Witness: $\{a^n b^n c^n\}$.

Context-sensitive ⊊ Turing-recognizable

TM recognize Type 0 languages. A_TM is recognizable but undecidable, whereas context-sensitive languages are decidable.

Final checklist • Examination strategy

Construction questions

Define state meanings, alphabet, start and accepting states; show complete transitions or productions; trace a sample.

Proof questions

State assumptions and quantifiers. For pumping, cover every legal split. For reductions, give direction and iff correctness.

Time management

Allocate time by marks; verify edge cases ε and malformed inputs; distinguish recognition from decision.

Course complete • 45 of 45 lectures

Regular languages

Finite automata, transducers, regular expressions, conversions, minimization, closure and pumping.

Context-free languages

Grammars, parse trees, ambiguity, normal forms, PDA equivalence, closure and pumping.

Computability and complexity

Turing machines, hierarchy, undecidability, Rice’s theorem, PCP, P/NP and reductions. Continue practicing constructions and proofs.